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Classification of multilayered folds based on harmonic analysis: example from central India

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Abstract—The harmony of multilayered folds can be determined objectively by comparing the b_3/b_1 (ratio of third and first coefficient of harmonic components of the Fourier series) for different surfaces in an individual fold. A plot of the b_3/b_1 value against surface number provides a simple graphical technique for assessing fold harmony. On such a diagram, a straight line parallel to abscissa denotes a strictly harmonic fold, while other lines and curves denote variation of fold shape from surface to surface. A new scheme of multilayered fold classification based on 'Index of Non-Harmony (INH)', which reveals the degree of variation in shape of different surfaces of the fold, is proposed. INH is obtained from standard deviation (σ_n) of b_3/b_1 ratios of 'n' number of surfaces of a quarter wave sector. On the basis of values of INH ($= 1000 \cdot \sigma_n$) the multilayered folds are classified as 'Strictly Harmonic' (INH = 0), 'Periharmonic' ($0 < \text{INH} \leq 15$), 'Subharmonic' ($15 < \text{INH} \leq 30$), 'Subnonharmonic' ($30 < \text{INH} \leq 45$), 'Nonharmonic' ($45 < \text{INH} \leq 75$) and 'Strongly Nonharmonic' (INH > 75). In a case study of polydeformed Precambrian rocks of central India, it is found that the index of nonharmony decreases in folds of later (younger) generations.

INTRODUCTION

Variation in the geometry of successive layers in multilayered folds can be determined with the help of dip isogons of Elliott (1965) or thickness parameters t'_α or T'_α of Ramsay (1967). Thus, individual folded layers can be classified as class 1A, 1B, 1C, 2 or 3 of Ramsay (1967) or class 1A1, 1A2, 1A3, 1B, 1C, 2, 3A, 3B or 3C of Zagorčev (1993). Twiss (1988) classified the shape of single folded surfaces on the basis of aspect ratio P, folding angle ϕ, and bluntness b. The shape of each folded surface can be described by harmonic (Fourier) analysis after Hudleston (1973).

Fourier analysis of fold surfaces defines the principal features of the fold shape with the help of first (b_1) and third (b_3) coefficients of the Fourier series. The analysis is very sensitive, and it records even the slightest change in the fold shape due to change in curvature or the relative position of the hinges of a folded layer (Hudleston, 1973; Ramsay and Huber, 1987). Hudleston (1973) gave a b_3 versus b_1 diagram in which the complex relation of b_3 and b_1 of the folded surfaces can be represented in a simplified way to obtain the fold shape and amplitude. On the basis of such analysis, Hudleston (1973) classified the fold shapes into 8 types. The classification was further developed by Singh and Gairola (1992) to include 16 types. However, these classifications do not specifically describe the degree of harmony of multilayered folds.

Multilayered folds are classified qualitatively as 'harmonic', in which fold surfaces exhibit the same wavelength and amplitude; and 'disharmonic' for the geometry exhibited by concentric folds or folds in which different layers exhibit different wavelengths and amplitudes (Price and Cosgrove, 1990). In the present paper, using the ratio b_3/b_1, a graphical scheme is suggested to represent the variation of shapes of folded surfaces in multilayered folds. Further, a new scheme of classification based on harmonic analysis is also proposed for such folds.

GRAPHICAL REPRESENTATION

The shape of successive surfaces in multilayered folds commonly varies. Variations in the shape are due to differing thicknesses and composition and the resulting mechanical anisotropy of successive layers. Variation of shapes among different surfaces of the layers may also be augmented by refolding or other post-folding deformations. Harmonic analysis is an accurate method to define the shape of a folded surface, but does not inherently describe variation in shapes of successive surfaces in a multilayered fold. For this purpose, a new graphical scheme (Fig. 2) based on harmonic analysis is suggested where the variation in shape across the profile of a multilayered fold can be viewed at a glance.

In the proposed graphical scheme (Fig. 2), the ordinate represents a ratio of third and first harmonic components of the Fourier series (b_3/b_1) and the abscissa represents the folded surface by their representive numbers as 1,2,...,n on a Cartesian plane. A line or curve on the b_3/b_1 versus n plots illustrates the consistency or degree of variation in the shape of successive surfaces. In this diagram, a straight line parallel to the abscissa denotes a strictly harmonic fold. In the folds where harmony is not maintained in successive surfaces, the parallelism of the line with the abscissa is lost, and thus, the patterns of the line or curve will represent the systematic or unsystematic variations. Variations in the shape of folded surfaces from one class to another can be recognised easily, if the ordinate is demarcated to show the boundaries of different classes of fold shapes as proposed by Hudleston (1973) or Singh and Gairola (1992) (Fig. 2) on the basis of b_3/b_1 values.

PROPOSED SCHEME OF CLASSIFICATION

If we look at the boundary shapes of fold classes of Hudleston (1973), we find that certain fold classes (e.g. box fold, semi-ellipse, parabola, sine-wave and chevron fold) are defined by fixed b_3/b_1 values, while the other classes accommodate a variable range of b_3/b_1 values. Singh and Gairola (1992) reduced the ranges by further introduction of eight additional classes in Hudleston's (1973) classification. The ranges could be further reduced by introducing more new classes. However, this will only increase the confusion in the nomenclature of fold shapes.

With the natural variability in the mineral composition, grain size and thickness of the successive layers in the rocks it is expected that disharmonic folds would be more common than harmonic folds because of variation in shape of the successive layers. Such variation in shape is reflected by different b_3/b_1 ratios. The proposed classification uses the standard deviation of b_3/b_1 values for n number of fold surfaces, which is obtained by following formula:

$$\sigma_n = \sqrt{\frac{(\Sigma x^2) - (\Sigma x)^2/n}{n}} \quad (1)$$

where $x = b_3/b_1$.

The index of nonharmony (INH) is obtained as follows:

$$INH = 1000 \cdot \sigma_n.$$

For a multilayered fold which has identical constitutive surfaces, the index of nonharmony (INH) is zero. Such a fold is called 'strictly harmonic'. The disharmony in folded surfaces increases with the increase of INH. In the proposed classification, a range of INH has been used to define the different harmonic classes of folds.

The boundary limits of each fold class of Hudleston (1973) and Singh and Gairola (1992) are shown along the ordinate in Fig. 2, which indicates that all the fold classes are not defined by standard ranges of b_3/b_1 values. Thus, for successive surfaces of a fold to remain in a particular pre-assigned fold class, the possible deviations from mean shape are also different. In order to obtain a normalised fluctuation (deviation) for all fold classes, the standard deviations (σ_n) of the boundary b_3/b_1 values of each fold class were obtained using equation 1. The σ_n value is zero for each of the five fold classes (i.e. box fold, semi-ellipse, parabola, sine wave and chevron fold). The normalised value of these deviations has been determined by taking the mean of all the 19 values of σ_n (see Appendix I) of the boundary limits of b_3/b_1 of the fold classes (excluding cuspate fold and multiple folds where lower and upper limits, respectively are not defined) of Singh and Gairola (1992). The normalised value thus obtained is nearly 0·015. Multiplied with 1000 (for simplification), this value (0·015) and its simple multiples have been used as INH in the present fold classification given in Table 1.

Table 1

INH	Fold Type
0.00	Strictly harmonic
$0.00 < INH \leq 15.00$	Periharmonic
$15.00 < INH \leq 30.00$	Subharmonic
$30.00 < INH \leq 45.00$	Subnonharmonic
$45.00 < INH \leq 75.00$	Nonharmonic
$INH > 75.00$	Strongly nonharmonic

Table 1 indicates that in all the fold classes except 'strictly harmonic', shape of the constitutive analysis of a multilayered fold are liable to change. However, the same range of INH may include different fold classes of Singh and Gairola (1992). Thus, the different fold classes in Table 1 provide an idea about the degree of consistency (harmony) of the shape of surfaces of a multilayered fold.

HARMONIC ANALYSIS OF FOLDS IN CENTRAL INDIA

As a case study, harmonic analysis has been carried out on 30 mesoscopic fold samples from multiply deformed (Roy and Bandyopadhyay, 1990; Banerji, 1991; Bhattacharya *et al.*, 1992) Precambrian rocks of central India. These folds are developed in low-grade metasedimentary slates and phyllites of the Mahakoshal Group and medium-grade schists and gneisses of the Chhotanagpur Granite Gneiss Complex (CGGC). The folds were recognised as F_1, F_2, F_3 and F_4 on the basis of detailed structural analysis carried out by the authors on the tectonites of southern Uttar Pradesh. It was found that the F_1, F_2, F_3 and F_4 exhibit tight-isoclinal, close, open and chevron or kink fold types, respectively (Fig. 1). The surfaces of the folds were traced on their profile section. In these folds, the upper surface of a layer serves as the lower surface of that lying above it. The hinge point (where the curvature is maximum; see Fig. 1) and the inflexion point (where the curvature changes sign; shown by dot at the end of the limbs in Fig. 1) were carefully located, as harmonic analysis is very sensitive to the location of these points (Hudleston, 1973; Ramsay and Huber, 1987) and the folds may offer appreciable changes in shape due to relative position of these points in adjacent surfaces (Fig. 2). One representative example of each, F_1, F_2, F_3 and F_4 folds from the Mahakoshal Group is shown in Fig. 1. The b_3/b_1 ratios of their constitutive surfaces (1,2,...,8) were calculated following Stabler (1968) and Hudleston (1973). The b_3/b_1 versus

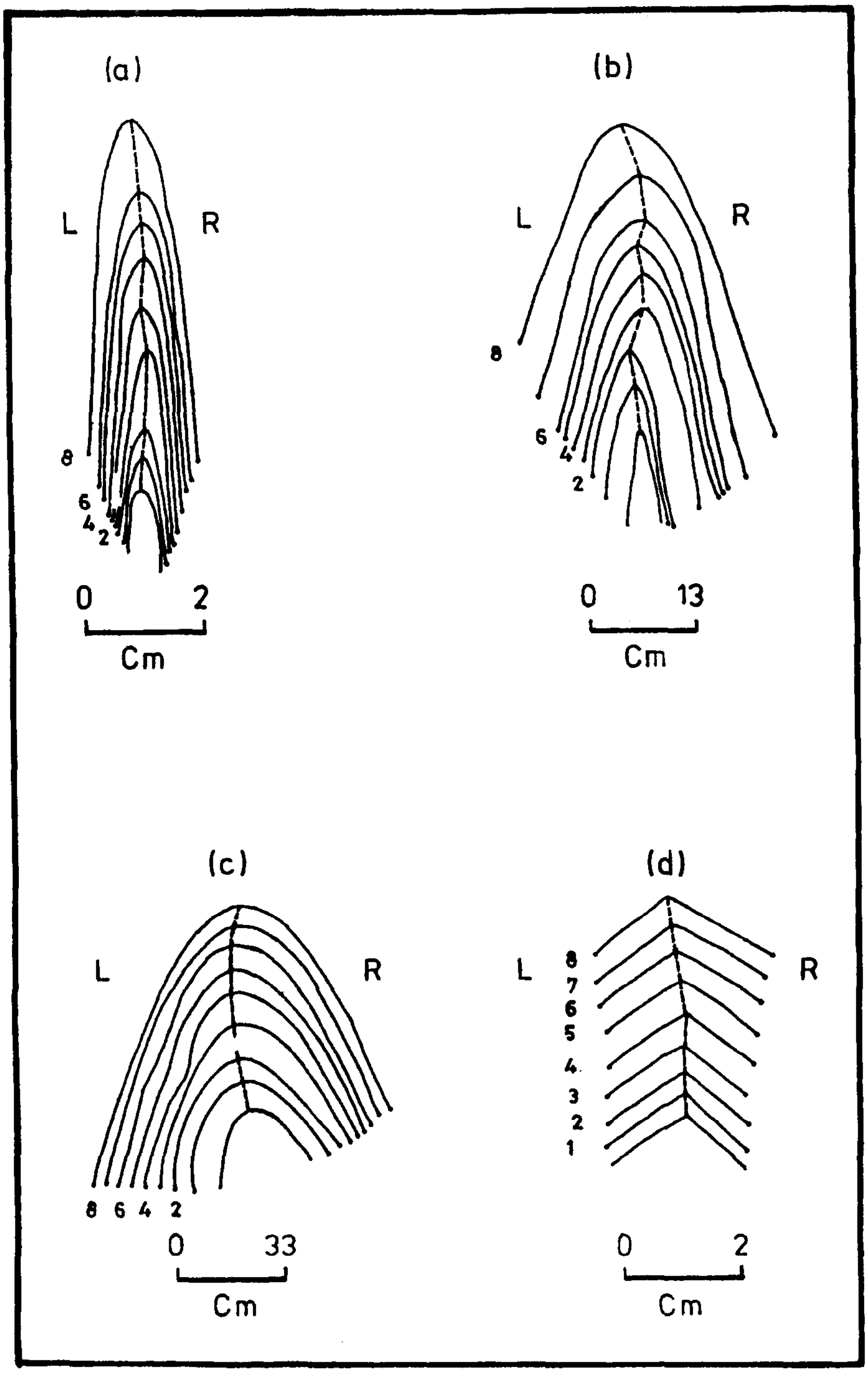

Fig. 1. Profiles of representative folds from four different generations of folds in the Mahakoshal Group which have been used to construct Fig. 2. (a) F_1 fold in slate (b) F_2 fold in slate (c) F_3 fold in slate and (d) F_4 fold in phyllite. L and R denote left and right quarter wave sectors of the folds, respectively. Broken lines connect the hinge points of the fold surfaces. Dots at the ends of each limb indicate the inflexion points.

n plots for the folds shown in Fig. 1 is presented in Fig. 2. It is evident from the figures that the folds show significant variation in their shapes.

The index of nonharmony (INH) obtained after harmonic analysis of a total of 93 quarter wave sectors (54 from Mahakoshal Group and 39 from CGGC) comprising of 4 to 8 layers, is presented in Fig. 3. In this diagram, the frequencies of different fold classes in different generations of folds are given along with the maximum, minimum and mean values of INH.

RESULTS AND DISCUSSION

Harmonic analysis of natural folds from the multiply deformed rocks of central India gives significant results

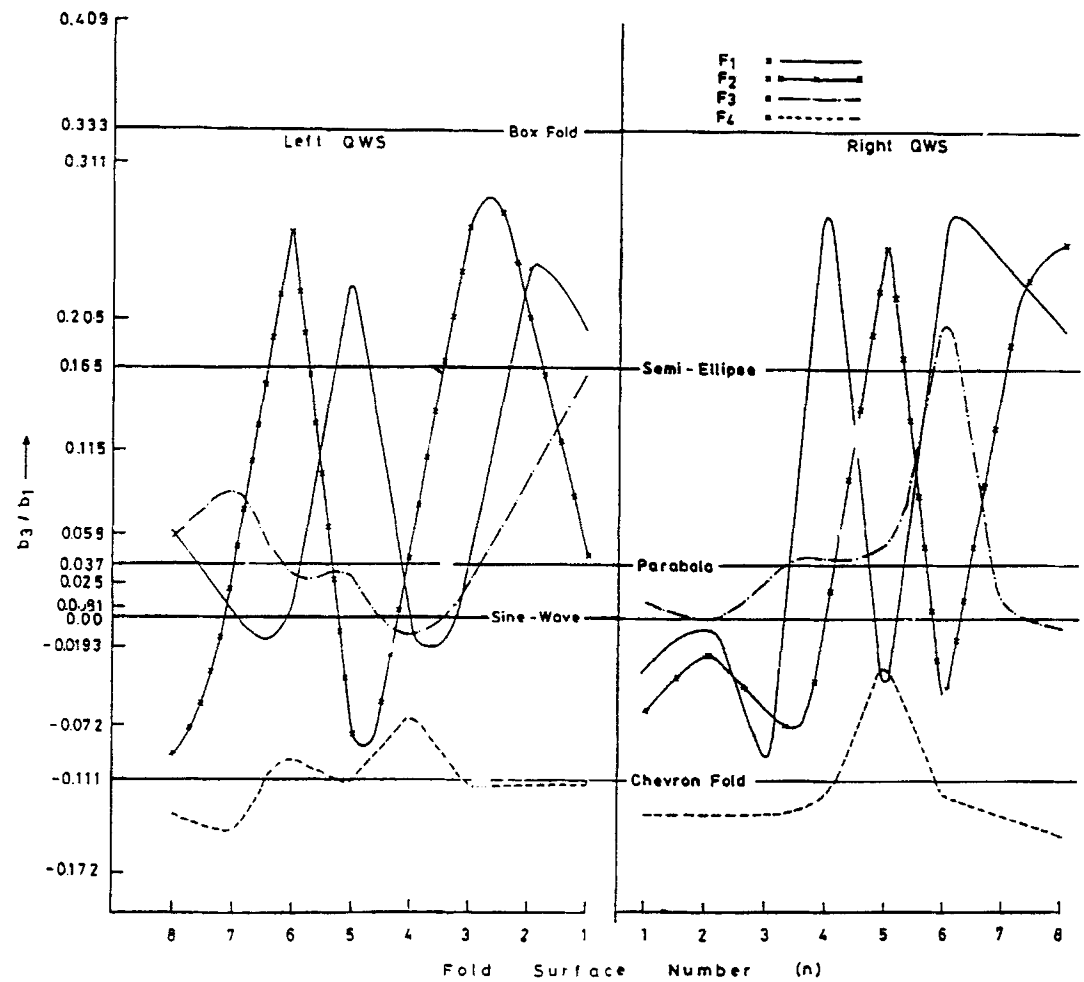

Fig. 2. The b_3/b_1 versus n plots for folds shown in Fig. 1. The fold classes given by Hudleston (1973) and Singh and Gairola (1992) are demarcated along the ordinate. The bold horizontal lines show five basic shapes defined by Hudleston (1973). Note that large fluctuation in fold shape is triggered by relative displacement of the hinge points (Fig. 1) in the successive fold surfaces even without significant change in curvature, which is particularly well marked in F_2 fold (Fig. 1b).

on the applicability of the proposed graphical representation and the classification scheme for multilayered folds. From the inspection of b_3/b_1 versus n graph (Fig. 2), it is evident that surfaces of earlier folds (F_1 and F_2) are characterised by greater variation in the b_3/b_1 ratios in a quarter wave sector whereas later folds (F_3 and F_4) have less variation in their b_3/b_1 ratios.

Results shown in Fig. 3 reveal that although there may be great fluctuations (large difference in minimum and maximum INH values) in some of the quarter wave sectors, the mean INH values for a group of quarter wave sectors of F_1, F_2, F_3 or F_4 folds are quite distinct. From the mean INH values of different fold generations (Fig. 3), it becomes evident that except for F_1 folds in the Mahakoshal Group (F_1 folds are obliterated in the CGGC), the folds of both tectonic units follow the expectation that provided other factors are the same, the younger folds should have greater harmony than the older ones. Thus, the greatest harmony is exhibited by F_4 folds (latest folds of schistosity) and the least by F_2 folds (first folds of schistosity), which are 'subharmonic' and 'strongly-nonharmonic', respectively. This is also evidenced by the leftward shifting (towards greater harmonic class) of the highest frequency class in the successively younger folds (Fig. 3).

The lesser harmony in F_1 folds (defined by mean INH) of the Mahakoshal Group than that in comparatively younger folds (Fig. 3) is perhaps due to the fact that F_1 folds have been recognised on the basis of patterns of bedding (S_1), while the F_2 and other younger folds have been recognised on the basis of schistosity (S_2), making a true comparision in these folds abstruse. The tight-isoclinal nature of F_1 folds may also be a factor for this anomaly.

CONCLUSIONS

Based on the harmonic analysis of multilayered folds in the present study, the following conclusions may be drawn.

(1) The proposed graphical representation of b_3/b_1 versus n (Fig. 2) shows the degree of harmony exhibited by fold surfaces in multilayered fold and simultaneously illustrates fold classes of Hudleston (1973) or Singh and Gairola (1992).

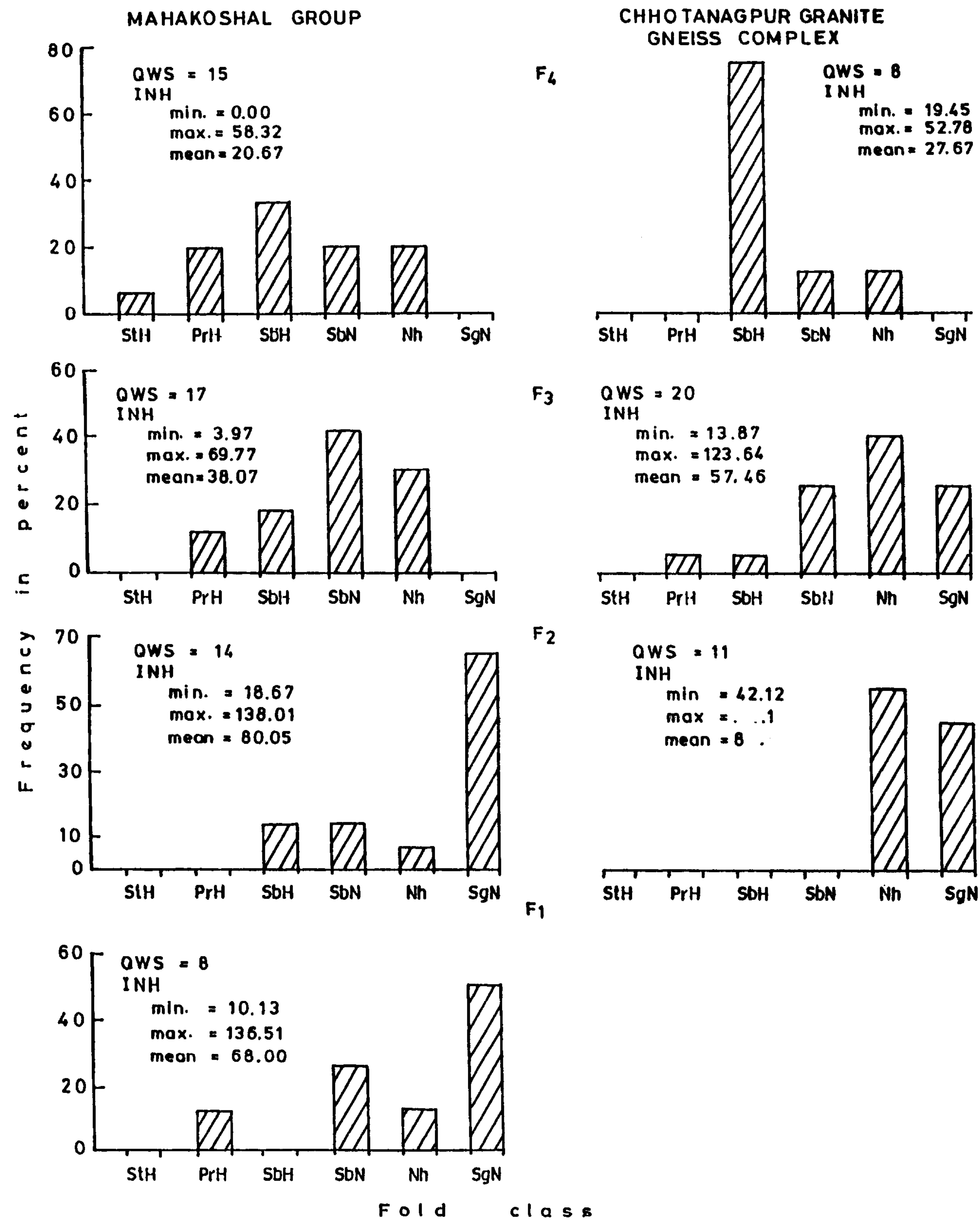

Fig. 3. Bar diagrams of the frequency of different fold classes in different generation of folds in the Mahakoshal Group and the Chhotanagpur Granite Gneiss Complex (CGGC). Abbreviations used: StH = Strictly harmonic folds (INH = 0), PrH = Periharmonic folds (0 < INH ≤ 15) SbH = Subharmonic folds (15 < INH ≤ 30), SbN = Subnonharmonic folds (30 < INH ≤ 45), Nh = Nonharmonic folds (45 < INH ≤ 75), SgN = Strongly nonharmonic folds (INH > 75), QWS = Quarter Wave Sector.

(2) The index of nonharmony (INH) quantifies the degree of fluctuation in shapes of folded surfaces.

(3) The INH classification (Table 1) can be applied on a multilayered fold specimen to express the degree of harmony in its constitutive surfaces.

(4) The mean INH value for a group of folds expresses the degree of consistency or inconsistency in shape of fold surfaces of any particular generation in an area.

(5) In two cases described here, folds of younger generations show greater harmony than those of the older generations.

This classification has some limitations. For example, the proposed classification is based on b_3/b_1 ratio and not on the actual wavelengths and amplitudes of the fold surfaces. Thus, theoretically, a fold possessing same shape but different wavelengths and amplitudes (e.g. concentric semi-circular folds), hence disharmonic, may offer a 'strictly harmonic' fold class because of constant b_3/b_1 ratio of the constitutive surfaces.

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APPENDIX

The limiting values of b_3/b_1 of different fold classes after Singh and Gairola (1992) are given in the following table. The standard deviation (σ_n) is obtained for the limiting values of b_3/b_1 for each fold class excluding multiple fold and cuspate folds for which the upper and lower limits respectively are not defined. The mean σ_n is also given in the following table.

Fold class	b_3/b_1		σ_n
	Upper limit	Lower limit	
Multiple Fold	—	0.409	—
Almost Box Fold	0.409	0.333	0.038
—do—	0.333	0.311	0.011
Box Fold	0.333	0.333	0.00
Between Box Fold and Semi-ellipse	0.311	0.205	0.053
Almost Semi-ellipse	0.205	0.165	0.02
—do—	0.165	0.115	0.025
Semi-ellipse	0.165	0.165	0.00
Between Semi-ellipse and Parabola	0.115	0.058	0.0285
Almost Parabola	0.058	0.037	0.0105
—do—	0.037	0.025	0.006
Parabola	0.037	0.037	0.00
Between Parabola and Sine Wave	0.025	0.0081	0.00845
Almost Sine Wave	0.0081	0.00	0.00405
—do—	0.00	−0.0193	0.00965
Sine Wave	0.00	0.00	0.00
Between Sine Wave and Chevron Fold	−0.0193	−0.072	0.02635
Almost Chevron Fold	−0.072	−0.111	0.0195
—do—	−0.111	−0.172	0.0305
Chevron Fold	−0.111	−0.111	0.00
Cuspate Fold	−0.172	—	—
	Mean σ_n		0.0152894